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METHOD OF PROBABILISTIC CONVOLUTIONS FOR INTERPRETING DATA OF ELECTROMAGNETIC LOGGING

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A new method is proposed for processing the data of electromagnetic logging in wells. The method is based on the hydrodynamic analysis of drilling mud filtrate penetration into the oil stratum, determination of the field of electrical resistances of the near-well zone with its subsequent averaging by convolutions with a kernel in the form of a logarithmically normal distribution of probe sensitivity.

Key words: *electromagnetic logging, immiscible filtration, salt exchange, oil saturation, drilling mud filtrate.*

Introduction. Devices for high-frequency isoparametric induction logging are used to determine physical parameters of oil strata. Information registered by focusing signals depends on the electrical resistance of a comparatively small near-well zone and undisturbed part of the oil stratum. In turn, the spatial distribution of resistance depends on complicated interrelated hydrodynamic and physicochemical processes inherent in oil-well drilling. Interpretation of logging data is a complicated problem whose solution often yields ambiguous results.

In electromagnetic logging of oil wells by devices of high-frequency isoparametric induction logging (HFIIL) or high-frequency electromagnetic logging (HFEL) [1, 2], probes with the following conditions on structural parameters are used:

$$\sqrt{f_i}L_i = \sqrt{3.5}\cdot 10^3, \qquad \Delta L_i/L_i = 0.2. \tag{1}$$

Here f_i [Hz] is the emission frequency, ΔL_i [m] is the distance between the measurement coils, and L_i [m] is the distance between the generator coil and the center of the remote measurement coil; $i = 1, 2, \ldots$ is the probe number.

Conditions (1) allow us to consider the medium response and, hence, confine ourselves to its measurement by each probe on an annular area of the oil formation surrounding the oil well. In fact, each probe measurement in an axisymmetric and vertically uniform oil stratum is a certain averaged electromagnetic characteristic of this area whose radius is denoted as r_i. With distance from it, the contributions of responses and the measurement accuracy of the ith probe decrease. The specific electrical resistances calculated by the phase difference of electromagnetic oscillations measured by the probes are called "apparent;" they can significantly differ from true resistances. We introduce the notation $x_i = r_i^2$ and $x = r^2$ for the squares of the characteristic and current radii, respectively. We assume that the "apparent" specific electrical resistances (SER) calculated on the basis of the measured data are most close to the true (ohmic) SER for $r = r_i$, i.e., correspond to some modes of the random distribution of measurement errors for the corresponding areas proportional to $x_i = r_i^2$. We also assume that the distribution functions ρ_i of sensitivity of each probe have the form of the logarithmically normal distribution

$$\rho_i = \frac{1}{2\sqrt{2\pi}}\,\frac{1}{\sigma_i x_i}\exp\Big(-\frac{\sigma_i^2}{2}\Big)\exp\Big(-\frac{1}{2\sigma_i^2}\,\ln^2\frac{x}{x_i}\Big). \tag{2}$$

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These functions have a maximum at the point $x = x_i$ and tend to zero as $r \to 0$ and $r \to \infty$. In (2), σ_i are the dispersions of deviations of measurements of apparent SER. Taking into account conditions (1) on the structural characteristics of HFIIL or HFEL instrumentation, we can assume that dispersion parameters σ_i for all probes are approximately identical and equal to σ, and the modes x_i are equal to their half-lengths squared. This assumption was confirmed by processing experimental data. The probability distribution density is normalized so that the following equality is satisfied:

$$\int_0^\infty \rho_i r\,dr = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma x_i} \exp\Big(-\frac{\sigma^2}{2}\Big) \exp\Big(-\frac{1}{2\sigma^2} \ln^2 \frac{x}{x_i}\Big)\,dx = 1.$$

In the proposed method of probability convolutions, the measured "apparent" SER are interpreted as the averaging of true electrical resistances of the medium with a weight function of the form (2). Thus, for an arbitrary true distribution $R(r)$, the ith probe registers the "apparent" SER averaged over the entire space, which can be calculated by the formula

$$\bar{R}_i = \int_0^\infty R(r)\rho_i(r) r\,dr. \tag{3}$$

Let the function of true SER have a stepwise form:

$$R = \begin{cases} R_n^0, & 0 < r_n, \\ R_0^0, & r_n < r < \infty. \end{cases} \tag{4}$$

Here r_n is the radius of the zone of penetration of the drilling mud filtrate into the aquifer with an initial resistance R_0^0 and R_n^0 is the resistance of the stratum completely saturated by the filtrate. The size of the penetration zone is determined from the water-mass balance equation

$$m\pi(r_n^2 - r_{\rm w}^2) = 2\pi r_{\rm w} \int_0^t V(r_{\rm w}, t)\,dt, \tag{5}$$

where $V(r_{\rm w}, t)$ is the velocity of filtrate penetration through the oil-well wall of radius $r = r_{\rm w}$ and m is the medium porosity. Substituting function (4) into formula (3) and integrating, we obtain

$$\bar{R}_i = \frac{R_n^0}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma} \ln\frac{x_n}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big] + \frac{R_0^0}{2}\Big[1 - \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma} \ln\frac{x_n}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big], \tag{6}$$

where the probability integral erf (x) is a known tabulated function and $x_n = r_n^2$.

Let us consider some particular cases.

1. Homogeneous oil stratum with a resistance R_0^0 without the penetration zone ($x_n = 0$). In this case, as $x_n \to 0$, from (6) we obtain erf $(-\infty) = -1$ and $\bar{R}_i = R_0^0$, i.e., the "apparent" resistance becomes the true resistance, and all probes register the true SER, independently of x_i.

2. The size of the penetration region is very large ($x_n \to \infty$). In this case, formula (6) again yields $\bar{R}_i = R_n^0$.

3. The penetration zone coincides with the zone of high sensitivity of the ith probe ($x_n = x_i$). This probe registers the "apparent" resistance calculated by the formula

$$\bar{R}_n = \frac{R_n^0 + R_0^0}{2} - \frac{R_n^0 - R_0^0}{2} \operatorname{erf}\Big(\frac{\sigma_n}{\sqrt{2}}\Big).$$

It follows from the above-said that an acceptable approximate mean arithmetic SER at the front of the drastic change in resistance is registered by a probe $i = n$ with a small dispersion parameter σ. The greater σ, the more underestimated the "apparent" SER as compared to the true value.

1. Filtrate Penetration into the Aquifer. The distribution of true resistances in the form (4) with the inequality $R_n^0 > R_0^0$ being satisfied corresponds to the simplest case of penetration of a comparatively fresh filtrate of the drilling mud into the stratum completely saturated by more mineralized water. In this kind of displacement, there is a distinct boundary between the solutions. Indeed, it was shown experimentally previously [3, 4] that the coefficient of filtration dispersion $D_{\rm f}$ in fine-grained sandstone can be represented as a sum of two components:

$$D_{\rm f} = \lambda_{\rm long}|v| + D_m m\psi.$$

Here λ_{long} is the coefficient of longitudinal dispersion, v is the velocity along the streamline, D_m is the diffusion coefficient in the free solution, and ψ is the tortuosity coefficient. In the experiments, we obtained $\lambda_{\text{long}} \approx 10^{-3}$ m and $\psi \approx 0.7$–0.8. In the case of short-time intrusions of the filtrate into the stratum, which corresponds to a real situation in drilling wells, the second term can be neglected. Then, for the coefficient of filtration dispersion, we obtain $D_{\text{f}} = \lambda_{\text{long}}|v|$. In the one-dimensional case, the length of the smearing zone of the initially sharp jump in mineralization, ΔR_{f}, can be estimated by the formula [5]

$$\Delta R_{\text{f}} = 2\sqrt{2D_{\text{f}}t/m},$$

where t is the time. Substituting the value of D_{f} into this expression and introducing the notation for the front coordinate $r_{\text{f}} \approx vt/m$, for $r_n = 0.5$ m, we obtain $\Delta R_{\text{f}} \approx 6.3 \cdot 10^{-2}$ m. In the axisymmetric case, other conditions being equal, the zone length is approximately equal to $3.6 \cdot 10^{-2}$ m, which is small as compared to the penetration depth. Therefore, the use of the piecewise-constant distribution of resistance in a stepwise form (4) is justified.

The calculations show that the "apparent" resistances measured by the probes in accordance with formula (3) are smoothed and coincide with the true resistances only close to the well and far from it.

2. Penetration into the Oil Stratum. In the case of immiscible displacement, we first consider the Buckley–Leverett scheme for two-phase filtration [6]. In the axisymmetric case, the laws of conservation of mass for oil and water

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + m\frac{\partial s}{\partial t} = 0, \qquad \frac{1}{r}\frac{\partial}{\partial r}(rv_1) - m\frac{\partial s}{\partial t} = 0$$

(s is the oil saturation and v and v_1 are the velocities of oil and water, respectively) yield the first integral

$$r(v + v_1) = r_{\text{w}}V(t)$$

and the hyperbolic equation for saturation

$$m\frac{\partial s}{\partial t} + \frac{r_{\text{w}}}{r}V(t)F'(s)\frac{\partial s}{\partial r} = 0. \tag{7}$$

Here F' is the derivative of the Buckley–Leverett function $F(s) = \alpha f(s)/[\alpha f(s) + f_1(s)]$, $f(s)$ and $f_1(s)$ are the relative phase permeabilities for oil and water, respectively, $\alpha = \mu_1/\mu$, and μ and μ_1 are oil and water viscosities. The function $F(s)$ has the properties $F(0) = 0$, $F(1) = 1$, $F'(s) \geqslant 0$ and in addition, by the flow properties $v = r_{\text{w}}V(t)F(s)/r$ and $v_1 = r_{\text{w}}V(t)[1 - F(s)]/r$ [$V(t)$ is the total velocity of the fluids]. For water penetration, we have $V(t) = v_1(r_{\text{w}}, t)$. Introducing the new variables

$$\tau = \frac{2}{mr_{\text{w}}}\int_0^t V(t)\,dt, \qquad \xi = \frac{x}{r_{\text{w}}^2} = \Big(\frac{r}{r_{\text{w}}}\Big)^2,$$

we can write an implicit solution of Eq. (7)

$$\xi = \tau F'(s) + 1$$

that satisfies the condition $r = r_{\text{w}}$ for $t = 0$. It follows from here that the variable τ is determined by the penetration depth (radius) r_n. In accordance with formula (5), we obtain the relation

$$\tau = (r_n/r_{\text{w}})^2 - 1. \tag{8}$$

At the front of water penetration $r = r_{\text{f}}$, oil saturation $s = s_{\text{f}}$ remains constant if the initial oil saturation s_0 of the oil stratum is constant [7]. Its value is determined from the transcendental equation

$$s_0 - s_{\text{f}} = [F(s_0) - F(s_{\text{f}})]/F'(s_{\text{f}}),$$

and the front $r = r_{\text{f}}$ propagates according to the law

$$r_{\text{f}} = r_{\text{w}}\sqrt{1 + \tau F'(s_{\text{f}})}.$$

In addition, it was shown [7] that the integral oil saturation averaged over the area $\langle s\rangle$ is independent of the position of the penetration front and is determined only by the constant s_{f}:

$$\langle s\rangle = s_{\text{f}} - F(s_{\text{f}})/F'(s_{\text{f}}).$$

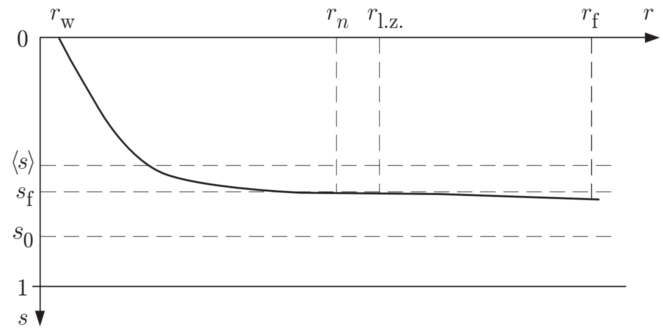

Fig. 1

Note, in contrast to the example considered in Sec. 1, here we have $r = r_n$, i.e., the radius of the total volume displacement of the fluids and the coordinate of the filtrate-penetration front $r = r_\mathrm{f}$ do not coincide. Since $(r_\mathrm{f}/r_\mathrm{w})^2 = \tau F'(s_\mathrm{f}) + 1$ and $F'(s_\mathrm{f}) > 1$ for $\alpha < 1$, then, with allowance for (8), we obtain

$$(r_\mathrm{f}/r_\mathrm{w})^2 = 1 + [(r_\mathrm{f}/r_\mathrm{w})^2 - 1]F'(s_\mathrm{f})$$

or

$$r_\mathrm{f}^2 - r_\mathrm{w}^2 = (r_n^2 - r_\mathrm{w}^2)F'(s_\mathrm{f}).$$

Since normally we have $F'(s_\mathrm{f}) > 1$, the radius of the penetration front is $r_\mathrm{f} > r_n$ (Fig. 1). In Fig. 1, we have $s_0 = 0.7$. The so-called low-resistivity zone following the penetration front r_f is identified during logging with the region of reduced specific electrical resistance. The salt concentration within this zone is higher than in the water filtrate and is almost the same as in native water. To prove this fact and estimate the size of the low-resistivity zone in the region $r_\mathrm{w} < r < r_\mathrm{f}$, we consider the area-averaged integral value of oil saturation $s = \langle s\rangle$. Figure 1 shows the dependence $s(r)$. The process of salt exchange in displacing oil or gas from the near-well zone by the drilling mud filtrate (in this case, water with a comparatively low content of salts c_p) in the stratum with natural water saturation $s_1 = 1 - s_0$ and some mineralization $c_0 > c_p$ is heterogeneous. During filtration, part of the oil phase immiscible with water remains in formation pores, and native water is usually little mobile with respect to the rock skeleton. In this case, in describing salt transport by the water phase, one should take into account the kinetics of mass exchange between heterogeneous structures containing mobile and localized water.

As was shown previously [8], in the one-dimensional case, an acceptable equation of salt-exchange kinetics is Freindlich's isotherm equation, which implies that the exchange intensity is proportional to the difference in salt concentrations in the mobile and localized water. If mass exchange occurs rather rapidly, the salt concentration in the low-resistivity zone formed behind the penetration front equals the salt concentration in native water. A similar result is obtained if the so-called method of active plates with the "plate height" tending to zero is used [9].

In axisymmetric motion, we consider two cases (Fig. 2). Case 1 corresponds to filtrate penetration with the front r_f without interaction with native water (Fig. 2a); case 2 corresponds to penetration to the same depth but with an infinitely rapid interaction in accordance with the mechanism described above (Fig. 2b). Figure 2 shows the mass-exchange schemes corresponding to these cases. The law of conservation of mass of salts dissolved in water yields the equation

$$(r_\mathrm{f}^2 - r_\mathrm{w}^2)[(1 - s_0)c_0 + (s_0 - \langle s\rangle)c_p] = (r_\mathrm{f}^2 - r_\mathrm{l.z.}^2)(1 - \langle s\rangle)c_0 + (r_\mathrm{l.z.}^2 - r_\mathrm{w}^2)(1 - \langle s\rangle)c_p.$$

Collecting the like terms, we obtain the main relation between the radius of the low-resistivity zone $r_\mathrm{l.z.}$, filtrate-penetration front r_f, initial oil saturation of the formation s_0, and mean oil saturation of the penetration zone $\langle s\rangle$:

$$r_\mathrm{l.z.}^2 = \frac{s_0 - \langle s\rangle}{1 - \langle s\rangle}\, r_\mathrm{f}^2 + r_\mathrm{w}^2\, \frac{1 - s_0}{1 - \langle s\rangle}.$$

The above-said is also valid in the case of filtrate penetration into a gas-bearing stratum. The displacement scheme is shown in Fig. 3 ($s_0 = 0.7$). In contrast to the oil-displacement scheme (see Fig. 1), here we observe

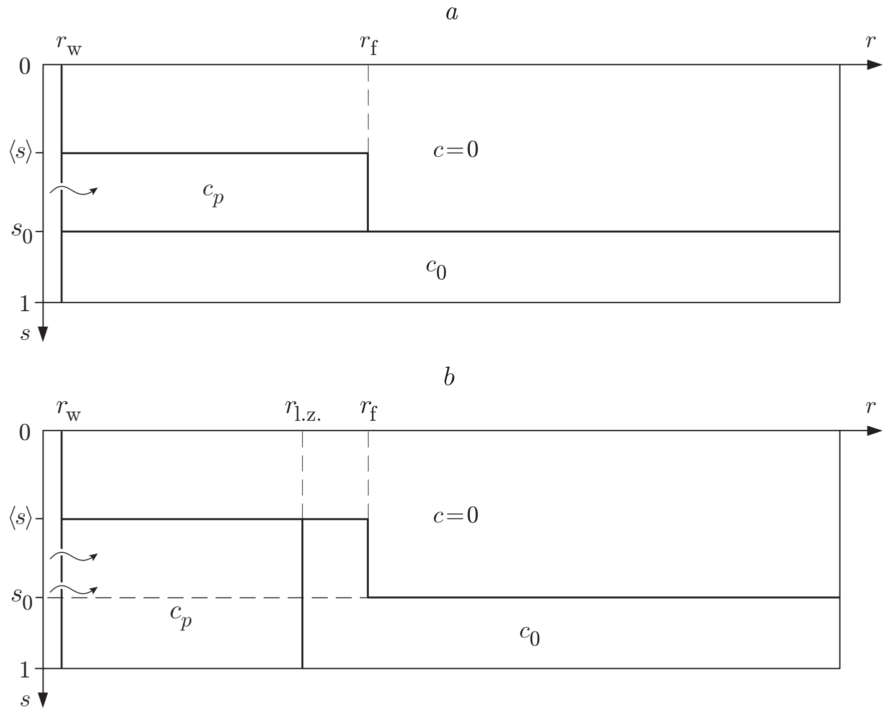

Fig. 2

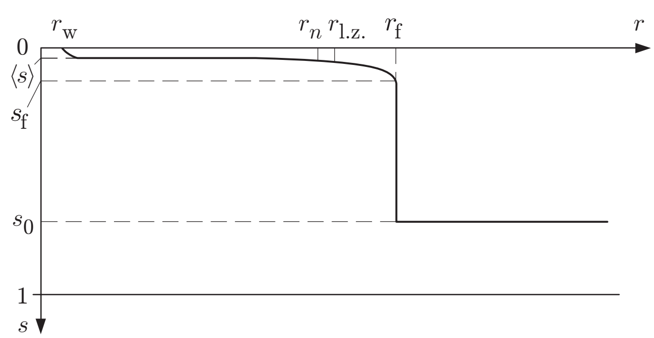

Fig. 3

low values of mean gas saturation. Thus, in mud-filtrate drilling of boreholes in oil- or gas-bearing formations, the function of electrical resistance in the near-well zone is represented as

$$R = \begin{cases} R_n, & r_{\rm w} < r \leqslant r_{\rm l.z.}, \\ R_{\rm l.z.}, & r_{\rm l.z.} < r < r_{\rm f}, \\ R_0, & r_{\rm f} < r < \infty. \end{cases}$$

Here $R_{\rm l.z.}$ is the specific resistance of the low-resistivity zone. Substituting these values into formula (3), we obtain

$$\bar{R}(r_i) = \frac{R_n - R_{\rm l.z.}}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\,\ln\frac{x_{\rm l.z.}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big]$$

$$+ \frac{R_{\rm l.z.}}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\,\ln\frac{x_{\rm f}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big] + \frac{R_0}{2}\Big[1 - \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\,\ln\frac{x_{\rm f}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big]. \qquad (9)$$

Note, Eqs. (6) and (9) for the "apparent" resistances registered by the ith probe can be considered as a generic expression for an arbitrary set of probes if they are characterized by identical dispersion and, in addition, satisfy the isoparametric conditions (1).

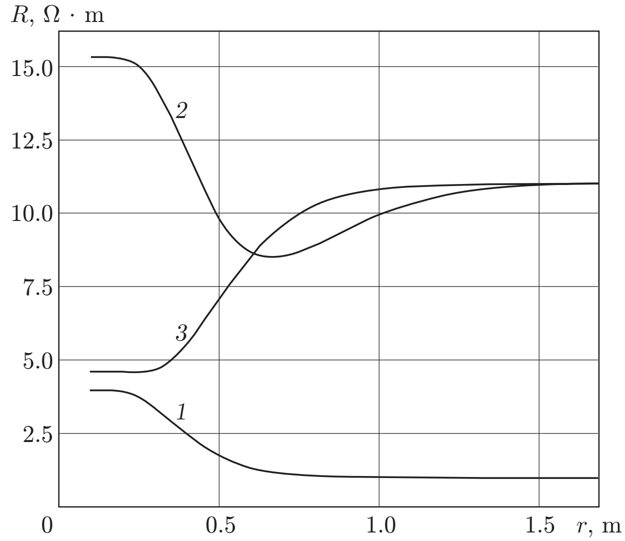

Fig. 4

3. Calculation Examples. We consider filtrate penetration into water-, oil-, and gas-bearing formations with identical penetration depths $r_n = 0.5$ m, identical concentrations and amounts of localized water $s_1^0 = 1 - s_0$, $s_0 = 0.7$ (for the oil-bearing stratum). For the aquifer, we assume that $R_n^0 = 4\ \Omega\cdot$ m and $R_0^0 = 1\ \Omega\cdot$m. In Fig. 4, this case corresponds to curve 1 obtained by formula (6).

For the oil-bearing stratum, we assume that $f(s) = s^{3.5}$, $f_1(s) = (1-s)^{3.5}$, and $\alpha = 0.16$. Then, for $s_0 = 0.7$, we obtain [7] $s_{\rm f} = 0.607$, $\langle s\rangle = 0.489$, $r_{\rm f} = 0.93$ m, and $r_{\rm l.z.} = 0.59$ m. In accordance with the known Archie law [10], the ratio of electrical resistance of a porous medium with complete saturation of porous space by an electrolyte to the resistance with incomplete saturation by the same electrolyte is inversely proportional to the saturation degree $s_1 = 1 - s$ with a certain coefficient $n \approx 2$. For the oil stratum considered, we can assume that $R_n = R_n^0/(1 - \langle s\rangle)^2$, $R_0 = R_0^0/(1 - s_0)^2$, and $R_{\rm l.z.} = R_0^0/(1 - \langle s\rangle)^2$. Then, using formula (9), we determine the dependence plotted by curve 2 in Fig. 4.

In the case of a gas-bearing bed, we can assume that $\alpha = 50$. The corresponding displacement scheme is shown in Fig. 3. The computations gave the following values of parameters: $s_{\rm f} = 0.12$, $F'(s_{\rm f}) = 1.6$, $\langle s\rangle = 0.07$, $x_{\rm f} = 0.27$, $x_{\rm l.z.} = 0.397$, $R_n = 4.62$, $R_{\rm l.z.} = 1.16$, and $R_0 = 11.1$. The dependence corresponding to these data is plotted by curve 3 in Fig. 4. In all examples considered, the dispersion σ was assumed to have a value of 0.68 obtained by solving inverse problems based on the logging data for aquifers.

It is seen in Fig. 4 that the logging curve for strata with medium oil saturation (curve 2) has a local minimum due to the presence of the low-resistivity zone. The research shows, that it is practically absent in strata with a rather high content of oil because of the low resolution of the probes at high values of σ. A similar picture is observed for gas-bearing formations: the presence of low-resistivity zones can be found only theoretically.

Conclusions. Based on the focusing properties of probes in HFIIL (5 probes) or HFEL (9 probes) devices, with the use of analytical solutions of problems of immiscible filtration with rapid salt exchange between the drilling mud filtrate and native water, simple engineering formulas suitable for interpreting logging charts are derived. The formulas obtained allow one to effectively solve inverse problems of determining physical properties of formations and fluids contained in them on the basis of electromagnetic logging data.

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